

Effect of light σ -meson Production in $p\bar{p} \rightarrow 3\pi^0$ at rest

Muneyuki ISHIDA, Toshihiko KOMADA*, Shin ISHIDA*
Taku ISHIDA**, Kunio TAKAMATSU*** and Tsuneaki TSURU**

*Department of Physics, Tokyo Institute of Technology
Tokyo 152-8551, Japan*

** Atomic Energy Research Institute, College of Science and Technology
Nihon University, Tokyo 101-0062, Japan*

*** KEK, Oho, Tsukuba, Ibaraki 305-0801, Japan*

**** Miyazaki U., Gakuen-Kibanadai, Miyazaki 889-2155, Japan*

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The $\pi^0\pi^0$ mass spectra and angular distributions around $K\bar{K}$ -threshold and at 1.5 GeV in $p\bar{p}(\text{at rest}) \rightarrow 3\pi^0$ in the Crystal Barrel experiment are reanalyzed by applying the new method, which is consistent with unitarity of S -matrix and expressed directly by resonance parameters. The effects of light σ -meson production are clearly seen to improve the fit with σ , in comparing with the fit without σ .

§1. Introduction

The light iso-singlet scalar σ meson plays an important role in the mechanism of spontaneous breaking of chiral symmetry, and to confirm its real existence is one of the most important topics in hadron physics. Recently in various $\pi\pi$ -production experiments^{1), 2)} a broad peak is observed in mass spectra below 1 GeV. Conventionally this peak was regarded as a mere non-resonant background, basing on the “universality argument,”³⁾ since no σ was seen in $\pi\pi$ scattering at that time. However, at present the $\pi\pi$ -scattering phase shift $\delta_S^{I=0}$ is reanalyzed by many authors^{4), 5), 1)} and the existence of σ meson is strongly suggested. A reason of missing σ in the conventional analysis is pointed out to be due to overlooking the cancellation mechanism,¹⁾ which is guaranteed by chiral symmetry, between the effects of σ and those of repulsive $\pi\pi$ -interaction. Moreover, the conventional treatment, based on the universality argument, of the low mass broad peak was shown not to be correct, and a new effective method, variant mass and width (VMW), is proposed to analyze resonance productions. In this method the production amplitude \mathcal{F} is directly represented by the sum of Breit-Wigner amplitudes with production couplings and phase factors (, including initial strong phases) of relevant resonances. The consistency of this method with the unitarity is seen from the following field theoretical viewpoint¹⁾.

Presently, after knowing the quark physics, the strong interaction \mathcal{L}_{str} among hadrons (in our example, mesons) is regarded as a residual interaction of QCD among color singlet $q\bar{q}$ -bound states, the “bare states,” denoted as $\pi = \bar{\pi}, \bar{\sigma}, \bar{f}_0$ and \bar{f}_2 . In switching off \mathcal{L}_{str} , the bare states appear as stable particles with zero widths. In switching on \mathcal{L}_{str} , they change into the physical states with finite widths. The unitarity of S -matrix is guaranteed automatically by the hermiticity of \mathcal{L}_{str} . The

VMW method is obtained directly as the representation of production amplitude by physical state bases, with a diagonal mass and width.

The VMW method is already applied¹⁾ to the analyses of pp -central collision $pp \rightarrow pp\pi^0\pi^0$ and $J/\psi \rightarrow \omega\pi\pi$ decay, leading to a strong evidence of existence of the light σ meson. In this paper we apply this method to analysis of the high statistics data on the process $p\bar{p} \rightarrow 3\pi^0$ at rest obtained in the Crystal Barrel experiment.²⁾

§2. Amplitude describing $p\bar{p} \rightarrow 3\pi^0$ at rest by VMW method

2.1. \mathcal{L}_{str} for $p\bar{p} \rightarrow 3\pi^0$ at rest

We apply the iso-bar model, describing the process in following two steps: In the first step the $p\bar{p}$ annihilates into the resonance f (f_0 and f_2) and π^0 , and the f decays into $2\pi^0$ in the second step. Since both p and \bar{p} are at rest, the relative momentum $p_\mu = p_{p\mu} - p_{\bar{p}\mu}$ (and the relative angular momentum $L_{p\bar{p}}$) between p and \bar{p} is 0. Charge conjugation parity P_C of $f\pi^0$ -system is +1. Thus the three types of (\bar{p}, p) bi-linear forms with $P_C = +1$ are possible: $\bar{p}i\gamma_5 p$, $\bar{p}i\gamma_5 \gamma_\mu p$ and $\bar{p}p$. The second type reduces to the first type by using the equation $-\partial_\mu(\bar{p}i\gamma_5 p) = 2m_p \bar{p}i\gamma_5 \gamma_\mu p + \bar{p}i\gamma_5 i\sigma_{\mu\nu} \overleftrightarrow{\partial}_\nu p = 2m_p \bar{p}i\gamma_5 \gamma_\mu p$ (, derived from Dirac equation and the above “rest condition”), and the third type is forbidden by parity. Thus only the first type remains.*¹⁾ The most simple form of $\mathcal{L}_{\text{str}}^{1,2}$ describing the 1st step $p\bar{p} \rightarrow f\pi^0$ and the 2nd step $f \rightarrow 2\pi^0$ is given, respectively, by

$$\begin{aligned}\mathcal{L}_{\text{str}}^1 &= \sum_{\bar{f}_0, \bar{f}_2} (\bar{\xi}_{\bar{f}_0} \bar{p}i\gamma_5 p \bar{f}_0 \pi^0 + \bar{\xi}_{\bar{f}_2} \bar{p}i\gamma_5 p \bar{f}_2 \partial_\mu \partial_\nu \pi^0), \\ \mathcal{L}_{\text{str}}^2 &= \sum_{\bar{f}_0, \bar{f}_2} (\bar{g}_{\bar{f}_0} \bar{f}_0 \pi^2 + \bar{g}_{\bar{f}_2} \bar{f}_2 \partial_{\mu\nu} (\pi \overleftrightarrow{\partial}_\mu \overleftrightarrow{\partial}_\nu \pi)).\end{aligned}\quad (2.1)$$

2.2. Amplitude by VMW method

First denoting the three π^0 as π_1, π_2 and π_3 with momenta p_1, p_2 and p_3 , respectively, we consider the π_1 and π_2 forming the resonance f with squared mass s_{12} (where $s_{ij} \equiv -(p_i + p_j)^2$) and with momentum $|\mathbf{p}|$ in z-direction. The π_1 has momentum $|\mathbf{q}|$ and polar angle θ in the f rest frame. In the lowest order in bare state representation⁶⁾ the amplitude is **)

$$2im_p f^{s_p s_{\bar{p}}} \left(\sum_{\bar{f}_0} \frac{\bar{\xi}_{\bar{f}_0} \bar{g}_{\bar{f}_0}}{\bar{m}_{\bar{f}_0}^2 - s_{12}} + \sum_{\bar{f}_2} \frac{\bar{\xi}_{\bar{f}_2} \bar{g}_{\bar{f}_2} N(s_{12}, \cos\theta_{12})}{\bar{m}_{\bar{f}_2}^2 - s_{12}} \right),$$

where

$$2im_p f^{s_p s_{\bar{p}}} \equiv \bar{p}(\mathbf{0}, s_{\bar{p}}) i\gamma_5 p(\mathbf{0}, s_p) \quad (s_{\bar{p}} \text{ and } s_p \text{ being spin of } \bar{p} \text{ and } p)$$

*¹⁾ This implies that the initial $p\bar{p}$ is in the 1S_0 state. However, in the original analysis²⁾, the phenomenological parameters related with the 3P_1 and 3P_2 states, which are considered not to contribute since of the above “rest condition,” are included.

**²⁾ $N(s_{12}, \cos\theta_{12})$ is obtained by the calculation of $N = -p_{3\mu} p_{3\nu} \mathcal{P}_{\mu\nu; \lambda\kappa} (p_1 - p_2)_\lambda (p_1 - p_2)_\kappa$, where we use the tensor projection operator $\mathcal{P}_{\mu\nu; \lambda\kappa}$ with mass squared s_{12} instead of $m_{f_2}^2$: $\mathcal{P}_{\mu\nu; \lambda\kappa} = \frac{1}{2}(\delta_{\mu\lambda} \delta_{\nu\kappa} + \delta_{\mu\kappa} \delta_{\nu\lambda}) - \frac{1}{3} \delta_{\mu\nu} \delta_{\lambda\kappa}$, where $\delta_{\mu\nu} = \delta_{\mu\nu} + \frac{P_\mu P_\nu}{s_{12}}$; $P_\mu = p_{1\mu} + p_{2\mu}$.

$$\begin{aligned}
f^{++} = f^{--} = 0, f^{+-} = -f^{-+} = -1. \\
N(s_{12}, \cos\theta_{12}) = -\frac{(s_{23} - s_{31})^2}{4} + \frac{16m_p^2 \mathbf{p}^2 \mathbf{q}^2}{3s_{12}}, \\
|\mathbf{p}| = \frac{\sqrt{(4m_p^2 - s_{12} - m_\pi^2)^2 - 4s_{12}m_\pi^2}}{4m_p}, \quad |\mathbf{q}| = \sqrt{\frac{s_{12}}{4} - m_\pi^2}
\end{aligned} \tag{2.2}$$

Owing to the effect of final (and initial) state interaction, the “full order” of the amplitude in physical state representation is given by

$$\begin{aligned}
A_{s_p s_{\bar{p}}}(s_{12}, \cos\theta_{12}) = 2im_p f^{s_p s_{\bar{p}}} \left(\sum_{f_0} \frac{r_{f_0} e^{i\theta_{f_0}}}{m_{f_0}^2 - s_{12} - i\sqrt{s_{12}}\Gamma_{f_0}(s_{12})} \right. \\
\left. + \sum_{f_2} \frac{r_{f_2} e^{i\theta_{f_2}} N(s_{12}, \cos\theta_{12})}{m_{f_2}^2 - s_{12} - i\sqrt{s_{12}}\Gamma_{f_2}(s_{12})} \right).
\end{aligned} \tag{2.3}$$

The symmetric amplitude, satisfying the statistics property of $3\pi^0$ system, $\mathcal{F}_{s_p s_{\bar{p}}}$ is obtained simply by its cyclic sum as

$$\mathcal{F}_{s_p s_{\bar{p}}} = A_{s_p s_{\bar{p}}}(s_{12}, \cos\theta_{12}) + A_{s_p s_{\bar{p}}}(s_{23}, \cos\theta_{23}) + A_{s_p s_{\bar{p}}}(s_{31}, \cos\theta_{31}). \tag{2.4}$$

Cross section is given by

$$d\sigma \sim \int_{2m_\pi}^{2m_p - m_\pi} d\sqrt{s_{12}} \frac{\sqrt{s_{12}}}{\pi} \frac{|\mathbf{p}|}{8\pi m_p} \frac{|\mathbf{q}|}{8\pi \sqrt{s_{12}}} \int_{-1}^1 d\cos\theta \overline{|\mathcal{F}|^2},$$

where

$$\begin{aligned}
\overline{|\mathcal{F}|^2} &\equiv (1/4) \sum_{s_p, s_{\bar{p}}} |\mathcal{F}_{s_p s_{\bar{p}}}|^2 = (1/2) |\mathcal{F}_{+-}|^2, \\
s_{12} + s_{23} + s_{31} &= 4m_p^2 + 3m_\pi^2, \\
s_{23} &= -4m_p(|\mathbf{p}||\mathbf{q}|/\sqrt{s_{12}})\cos\theta + m_\pi^2 + 2m_p E_3, \\
E_3 &= \sqrt{m_\pi^2 + \mathbf{p}^2} = (4m_p^2 - s_{12} + m_\pi^2)/4m_p.
\end{aligned} \tag{2.5}$$

§3. Results of Analysis

We analyze the experimental data, the $\pi^0\pi^0$ mass spectra and angular distributions around $K\bar{K}$ -threshold and at 1.5 GeV, which are published in the paper²⁾ by Crystal Barrel collaboration, using our formulas Eq.(2.5). We take into consideration as the physical particles $f_0 = \sigma, f_0(980), f_0(1370), f_0(1500)$, and $f_2 = f_2(1275), f_2(1565)$.

We take into account the $\pi\pi$ and $K\bar{K}$ couplings of the relevant resonances, where we consider the effects of all the inelastic channels are represented by the $K\bar{K}$ coupling.

The result of the fit is shown in Fig. 1.

The mass and width of σ obtained are $m_\sigma = 540^{+36}_{-29}\text{MeV}$ and $\Gamma_\sigma = 385^{+64}_{-80}\text{MeV}$ (error corresponding to the 5σ -deviation). The reduced χ^2 is given by $746.9/(281 -$

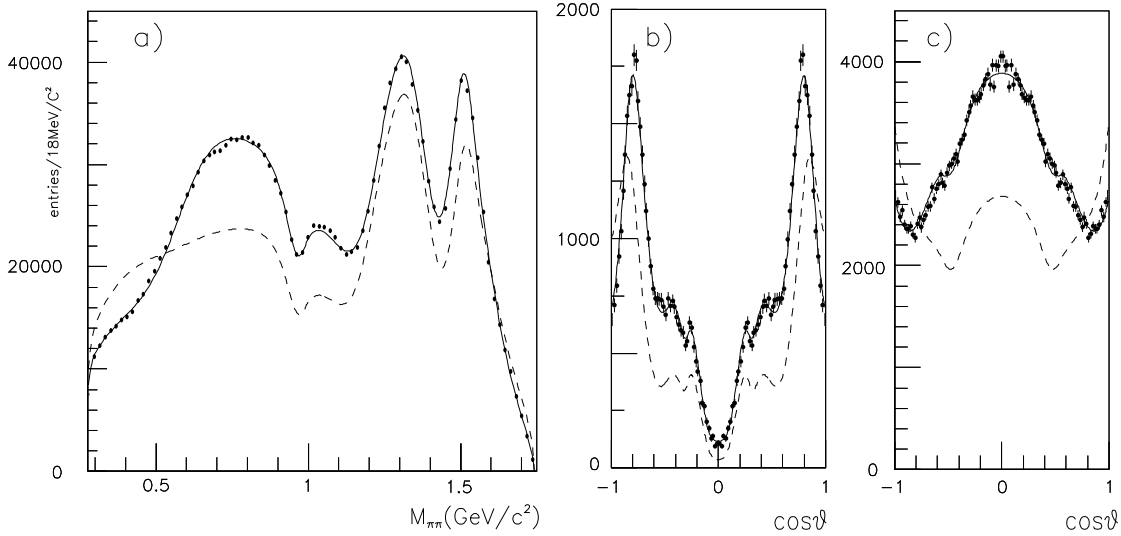


Fig. 1. Result of the fit by the amplitude represented in terms of the resonances with $\pi\pi$ and $K\bar{K}$ couplings: (a) $\pi^0\pi^0$ mass distribution, and $\cos\theta$ distributions (b) around $K\bar{K}$ -threshold and (c) at 1.5 GeV. Theoretical curves of the $\cos\theta$ distributions around $K\bar{K}$ -threshold and at 1.5 GeV, are given at the energies, 995.3 MeV and 1500 MeV, respectively. The total χ^2 is $\chi^2_{\text{total}} = 746.8$, as a sum of the respective contributions from (a), (b) and (c), $406.5 + 148.1 + 192.2$. The reduced χ^2 is $\chi^2_{\text{total}}/(N_{\text{data}} - N_{\text{param}}) = 746.9/(281 - 30) = 2.98$. Dashed curves represent the spectra obtained by setting $r_\sigma = 0$ in the fit.

30) = 2.98. Respective contributions to the χ^2 from the mass and angular distributions(a.d.) around $K\bar{K}$ and at 1.5GeV are 406.5(mass), 148.1(a.d. around $K\bar{K}$) and 192.2(a.d. at 1.5GeV).*) **) The properties of all the relevant resonances used in the fit are given in Table 1.

We have also tried the fit without considering the $K\bar{K}$ couplings of the resonances. The result of our fit with the $K\bar{K}$ couplings is almost the same as the one without the $K\bar{K}$ couplings; and in order to determine the $K\bar{K}$ couplings and the other ones, it is necessary to analyze the data on the corresponding channels directly.

In order to see the effect of σ -meson production in our fit, in Fig. 1 the spectra given by setting $r_\sigma = 0$ are also given by dashed lines. Effect of σ -production is seen to be crucially important in reproducing the structure of mass spectra below 1 GeV.

We have also tried the fit without introducing the σ Breit-Wigner amplitude.

*) The obtained χ^2 value may be compared with the one by the original fit ²⁾: We have estimated their corresponding χ^2 by reading the deviations of their fit from the experimental points and obtained the numerical values as $\chi^2/(N_{\text{data}} - N_{\text{param}}) = 835.4/(282 - 25) = 3.25$. Respective contributions to the χ^2 are 432.8(mass), 113.4(a.d. around $K\bar{K}$) and 289.3(a.d. at 1.5GeV). Almost the similar but slightly improved fit is obtained in the present VMW method. However, note that their parameters were determined by analyzing the original two-dimensional Dalitz plot directly. They reported that χ^2 in this case is $\chi^2/N_F = 2028/(1338 - 34) = 1.6$.

**) Number of data points in the figures given in ref.2) is $282 = 82(\text{mass}) + 100(\text{a.d. around } K\bar{K}) + 100(\text{a.d. at 1.5GeV})$. In our analysis the first one point close to the threshold of mass spectra is removed since this point is at 279MeV, where $\pi^0\pi^0$ channel is open but $\pi^+\pi^-$ channel is almost closed, and accordingly a special treatment of $\pi\pi$ widths of resonances is required.

Table I. Values of the resonance parameters obtained by the χ^2 fit with both $\pi\pi$ and $K\bar{K}$ couplings($g_{\pi\pi}$, $g_{K\bar{K}}$). The errors correspond to 5 σ deviation. The mark * represent that the corresponding values are not able to be determined by the fit: The $g_{K\bar{K}}$ (and correspondingly $\Gamma_{K\bar{K}}$) of $f_2(1270)$ is insensitive to the fit. The mass of $f_2(1270)$ falls in its constrained lower limit, and the $\pi\pi(K\bar{K})$ coupling of $f_0(980)$ does in its upper(lower) limit. In our analysis energy-dependent widths of the resonances are given by the sum of the $\pi\pi$ and $K\bar{K}$ widths, $\Gamma(s) = \Gamma_{\pi\pi}(s) + \Gamma_{K\bar{K}}(s)$, where $\Gamma_i(s) \equiv \frac{g_i^2 |\mathbf{p}_1|}{8\pi s}$ ($i = \pi\pi, K\bar{K}$) for scalars and $\Gamma_i(s) \equiv \frac{4g_i^2 |\mathbf{p}_1|^5}{15\pi s^3}$ ($i = \pi\pi, K\bar{K}$) for tensors. The values of Γ in the table are the widths at the respective peak positions of the resonances: $\Gamma \equiv \Gamma(s = m_{\text{res}}^2)$, except for the $\Gamma_{K\bar{K}}$ of $f_0(980)$ (, of which value is calculated as $\Gamma_{K\bar{K}} \equiv \frac{1}{N} \int_{4m_\pi^2}^{\infty} ds \Gamma_{K\bar{K}}(s) \frac{1}{(m_{\text{res}}^2 - s)^2 + s\Gamma_{\text{tot}}(s)^2}$; $N \equiv \int_{4m_\pi^2}^{\infty} ds \frac{1}{(m_{\text{res}}^2 - s)^2 + s\Gamma_{\text{tot}}(s)^2}$). The relative ratios of the absolute magnitude of angular distributions compared with the mass spectra are, respectively, 0.0695 and 0.1606. The central values of production coupling r and of production phase $\theta(\text{deg.})$ are given, respectively, by $(r_\sigma, r_{f_0(980)}, r_{f_0(1370)}, r_{f_0(1500)}, r_{f_2(1270)}, r_{f_2(1565)}) = (370, 108, 1720, 676, 1721, 4724)$ and $(\theta_{f_0(980)}, \theta_{f_0(1370)}, \theta_{f_0(1500)}, \theta_{f_2(1270)}, \theta_{f_2(1565)}) = (258, 164, 111, 276, 71)$.

	mass(MeV)	$g_{\pi\pi}(\text{GeV})$	$\Gamma_{\pi\pi}(\text{MeV})$	$g_{K\bar{K}}(\text{GeV})$	$\Gamma_{K\bar{K}}(\text{MeV})$
σ	540_{-29}^{+36}	$3.47_{-0.38}^{+0.28}$	385_{-80}^{+64}		
$f_0(980)$	964_{-10}^{+11}	2.25	100	2	—
$f_0(1370)$	1368_{-16}^{+19}	$4.10_{-*}^{+0.30}$	240_{-*}^{+36}	*	*
$f_0(1500)$	1518_{-17}^{+12}	$1.90_{-0.49}^{+0.94}$	46_{-21}^{+58}	$2.65_{-*}^{+1.01}$	70_{-*}^{+63}
$f_2(1270)$	1220	$8.60_{-0.63}^{+1.00}$	142_{-20}^{+35}	16	38
$f_2(1565)$	1552_{-21}^{+15}	$7.88_{-*}^{+3.90}$	98_{-*}^{+121}	$14.7_{-*}^{+3.5}$	100_{-*}^{+54}

In this case the broad peak structure below 1 GeV in the mass spectra are only roughly reproduced by the combinatorial background coming from the higher mass $2\pi^0$ resonances due to the statistics property of $3\pi^0$ system. The corresponding χ^2 is $3112/(281-26)=12.2$, which is much worse than our best fit with σ meson. This seems to give a strong evidence for σ -existence.

§4. Comparison with other analyses

Several extensive analyses (including the original one²⁾) on the relevant experimental data by Crystal Barrel collaboration have been thus far done. However, all the analyses seem to be done, more or less, under the influence of “universality argument³⁾”.

According to this argument, all the $\pi\pi$ production amplitudes \mathcal{F} must be parametrized through the “universal” $\pi\pi$ scattering amplitude \mathcal{T} as

$$\mathcal{F} = \alpha(s)\mathcal{T}, \quad (4.1)$$

with slowly varying real function $\alpha(s)$ which corresponds to the $\pi\pi$ production couplings of the production channel and is process-dependent. This equation is believed to be based on the unitarity or final state interaction(FSI) theorem.

However, it has been pointed out that, in order to apply the FSI theorem presently, after knowing quark physics, we must make a special attention⁶⁾ on the bases of representation of \mathcal{T} and \mathcal{F} . Before knowing quark physics, the S -matrix

of strong interaction S_{str} was represented in terms of only stable particles such as π and N . However, now S_{str} should be described by the interaction Hamiltonian $\mathcal{H}_{I,\text{str}}$ among the bare states, the color neutral stable bound states of quarks and/or antiquarks. The observed $\pi\pi$ resonances, such as σ or $f_0(980)$, and all the resonant states must be treated equally to the stable states, such as one pion or two pion states, as complete set describing the hadron world. They have mutually independent $\pi\pi$ (and/or $K\bar{K}$) production couplings, in principle. Accordingly, Eq.(4-1) in its original form has proved not to be correct.

In the pioneering work⁷⁾ by Aker et al. of Crystal Barrel(CB) collaboration, the $I = 0$ S wave production amplitude $\mathcal{F}_S(s_{12})$ (in our notation) is taken as

$$\mathcal{F}_S(s_{12}) = \alpha \frac{1}{\rho_1(s_{12})} e^{i\delta_S^{I=0}(s_{12})} \sin \delta_S^{I=0}(s_{12}) = \alpha \mathcal{T} \quad (4.2)$$

with the constant $\pi\pi$ coupling α , basing on the original “universality argument” Eq.(4-1). The symmetric production amplitude is obtained by taking the cyclic sum as Eq.(2-4), while the scattering amplitude \mathcal{T} is picked up from the reference by Au, Morgan, Pennington(AMP) in 3), where the conventional \mathcal{K} matrix analysis was done for the CERN-Munich $\pi\pi$ scattering phase shift $\delta_S^{I=0}$.⁸⁾

In a series of works by Anisovich, Sarantsev, Bugg and Zou et al.⁹⁾⁻¹¹⁾ being done in the line of this thought⁷⁾, the relation of “the universality argument” $\mathcal{F} = \alpha(s)\mathcal{T}$ with real $\alpha(s)$ function is argued to be not correct, since of the possible effect of the strong phases(basing on the detailed consideration of their origins, for example, triangle singularities). They used N/D formalism and mentioned that in production processes the complex N function $N'(s)$, being independent of $\pi\pi$ scattering N function $N(s)$, is necessary. A great variety of modification is allowed for the parametrization of $N'(s)$, and they try to fit the spectra of $p\bar{p}$ annihilation with three tentative forms of $N'(s)$ with complex couplings. Here, they introduce the scalar($f_0(1365)$ and $f_0(1520)$) and tensor($f_2(1270)$ and $f_2(1560)$) Breit-Wigner amplitudes above 1.1 GeV with complex production couplings. In our interpretation, their analysis is, so to speak, two-fold: Below 1.1 GeV it was done without taking into account of the freedom of production couplings of resonances by following the viewpoint of “the universality argument,” while above 1.1 GeV their method is equivalent to the VMW method, where this freedom is explicitly introduced.

In all the above analyses the pole positions of \mathcal{F} matrix in low energy region below ~ 1 GeV are determined only through the analyses of CERN-Munich $\delta_S^{I=0}$ ⁸⁾ by \mathcal{K} matrix method, although there are many varieties of parametrization methods of \mathcal{K} . This is also the case in the original analysis²⁾ by Amsler et al. of CB collaboration. In this analysis the $\pi\pi$ scattering and production amplitudes are given, respectively, in the \mathcal{K} matrix representation as: $\mathcal{T} = \mathcal{K}/(1 - i\rho\mathcal{K})$, $\mathcal{F} = \mathcal{P}/(1 - i\rho\mathcal{K})$, where \mathcal{K} -matrix and \mathcal{P} -matrix are taken in pole-dominative form;

$$\mathcal{K} = \sum_{\alpha} \frac{g_{\alpha}^2}{(m_{\alpha}^2 - s)} + c_{\text{BG}}, \quad \mathcal{P} = \sum_{\alpha} \frac{e^{i\theta_{\alpha}} \xi_{\alpha} g_{\alpha}}{(m_{\alpha}^2 - s)}. \quad (4.3)$$

Here the summation α is taken for the \mathcal{K} -matrix states, which are related⁶⁾ to the physical states corresponding to the poles of \mathcal{T} (or \mathcal{F}). In the case with no

production phases, $\theta_\alpha = 0$, the \mathcal{F} and \mathcal{T} have the same phase, coming from the common factor $1/(1 - i\rho\mathcal{K})$, and the final state interaction theorem is satisfied. Actually in their analysis this phase was set to be the experimental scattering phase shift $\delta_S^{I=0}$. However, in the above \mathcal{K} -matrix parametrization, when s is close to m_α^2 , \mathcal{K} diverges and the phase must take the value $90^\circ (+n \times 180^\circ)$. This gives a very strong constraint for the value of m_α . The experimental $\delta_S^{I=0}$ passes through 90° at about $\sqrt{s} \simeq 900$ MeV, and so the m_α becomes $m_\alpha \simeq 900$ MeV, which is much larger than $m_\sigma (\simeq 600 \text{ MeV})$. Thus, no existence of light σ is implicitly assumed from the beginning. This situation is common in all the analyses^{7), 9) - 11), 2)} thus far made.

In our method, the $\delta_S^{I=0}$ is analyzed by introducing the repulsive background phase shift δ_{BG} , which is required from chiral symmetry.^{12), 13), 1)} The scattering S -matrix and correspondingly the \mathcal{K} -matrix are parametrized by

$$S = S^{\text{Res}} S^{\text{BG}}, \quad \mathcal{K} = \frac{\mathcal{K}^{\text{Res}} + \mathcal{K}_{\text{BG}}}{1 - \rho^2 \mathcal{K}^{\text{Res}} \mathcal{K}_{\text{BG}}}. \quad (4.4)$$

The \mathcal{K}^{Res} in denominator of \mathcal{K} removes the poles of $\mathcal{K}^{\text{Res}} = \sum_\alpha g_\alpha^2 / (m_\alpha^2 - s)$ in the numerator in the total \mathcal{K} -matrix and we can take the light $m_\alpha \simeq 600 \text{ MeV}$.^{*)}

Correspondingly, the \mathcal{F} is represented by ^{**)}

$$\mathcal{F} = \frac{\mathcal{P}^{\text{Res}}}{1 - i\rho\mathcal{K}^{\text{Res}}} e^{i\delta_{\text{BG}}}, \quad (4.5)$$

which satisfies the final state interaction theorem. This \mathcal{F} is able to be⁶⁾ rewritten into the form, applied in VMW method, in the physical state representation. In our approach, whether the light σ meson exists or not is determined directly from the experimental data themselves, as was done in §3.

§5. Conclusion

Through the results of analyses given above we may conclude that the effects of production of light σ meson are clearly shown. The numerical values of mass and width of σ are obtained as $m_\sigma = 540^{+36}_{-29} \text{ MeV}$, $\Gamma_\sigma = 385^{+64}_{-80} \text{ MeV}$, which are consistent with those obtained in our phase shift analysis⁵⁾ ($(m_\sigma, \Gamma_\sigma) = (535 \sim 675, 385 \pm 70) \text{ MeV}$). However, the effect of σ in this process is, in principle, not able to be discriminated from those of higher mass resonances, such as $f_0(1370)$, due to the effects coming from statistics property of $3\pi^0$ system. In order to avoid this, it is desirable to analyze also the process, $\bar{p}n \rightarrow \pi^0 \pi^0 \pi^-$, through the similar method.

Finally it should be noted that the experimental data of the spectra applied in this paper were obtained by reading out the corresponding figures of ref. 2) and incomplete. The excellent reproduction of the data encourages us to study more in details. It is desirable to reanalyze directly the experimental data of the Dalitz plot.

^{*)} On the other hand, the background matrix, c_{BG} , in the conventional \mathcal{K} -matrix cannot describe the global phase motion corresponding to δ_{BG} in our method, and the small value of m_α is not permissible.

^{**)} Here we neglect the possible effect of non-resonant $3\pi^0$ production.

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